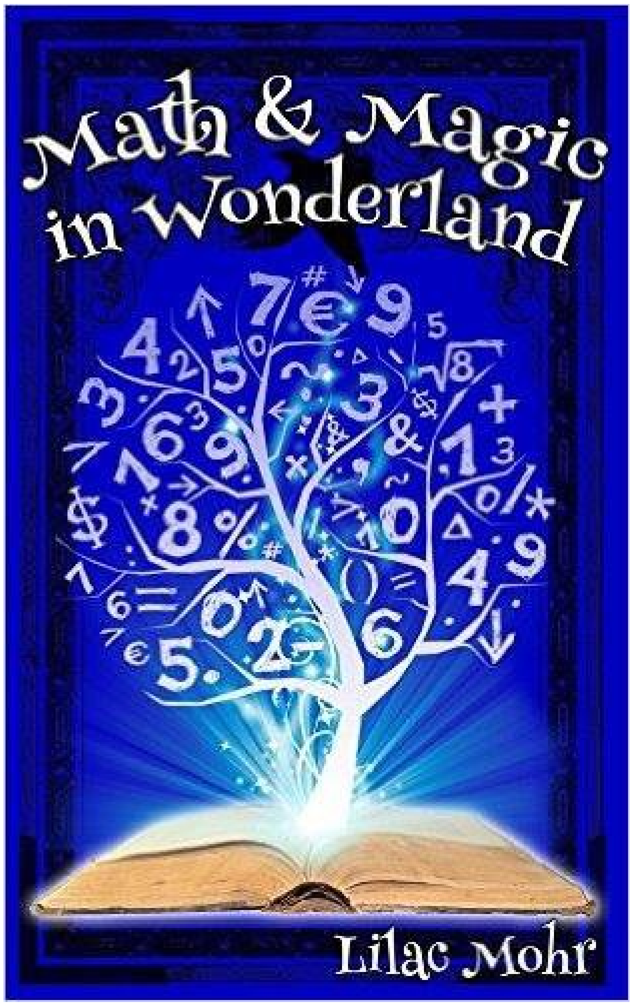
Math and Magic in Nature

This PDF contains a set of lessons that explore the natural science themes found in the math adventure novel "Math and Magic in Wonderland":



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Introduction

Dear reader,

When I set out to write a novel full of classic math and logic puzzles, I was completely focused on the field of mathematics. As a software engineer with an advanced degree in Statistics, I felt most at home immersed in a world of numbers, computer code, and mathematical proofs. It took a set of twins (Elizabeth and Lulu, the characters in the book) to open my eyes to the beauty of math in the world all around us. As I developed these characters, they grew from a set of brainy girls into unique individuals who enjoy observing their surrounding and making connections between many different fields – math, science, art, and history. The story soon developed into much more than a math book. I planted seeds throughout the novel to inspire readers to follow their own "rabbit trails" and investigate relationships between math and the real world. In Lulu's words:

"The more mathematical patterns we uncover in our world,
the more reason we have to fill our hearts with awe, reverence, and wonder."

I am very excited to bring you this special companion to "*Math and Magic in Wonderland"*, in which we'll explore the connection between math and nature. I've chosen a variety of topics inspired by the novel. Instead of a comprehensive science curriculum, these investigations are meant to spark your curiosity and, just like the book, they can be enjoyed on many levels across a large span of age groups. Be sure to follow your own "rabbit trails" and have fun!

Sincerely,

Lilac Mohr
Author of "*Math and Magic in Wonderland*"

Suggested schedule

The science activities in this guide are meant to be explored as you read through each chapter of the novel, but feel free to set your own schedule. If you'd like to focus on a single chapter each week, this sample schedule is provided for reference:

Weekly schedule

1. Monday: Read a chapter of "*Math and Magic in Wonderland"* and solve the problems found within the text of the chapter.

2. Tuesday: Solve the math, language, and logic problems found in the "Play Along" section at the end of the chapter.

3. Wednesday: Explore the videos and additional activities for the chapter in the Math and Magic Book Club found at: http://learnersinbloom.blogspot.com/p/blog-page_3.html

4. Thursday: Read and complete the related sections in this companion guide to learn more about natural science topics from the chapter.

5. Friday: If you did not finish all of this chapter's science activities on Thursday, complete them now. Otherwise, ask questions, follow rabbit trails, and perform your own investigations.

Chapter 1: Mrs. Magpie's Manual

Topic 1.1: Animal life expectancy

In Chapter 1, Lulu and Elizabeth solved their first puzzle: calculating Mrs. Magpie's age. When biologists discuss the lifespan of a species, they are not talking about the age of an individual animal. Instead, average lifespan, or life expectancy, refers to the average age that members of that species can be expected to live under optimal conditions (where they have enough food and the right climate).

> Think about it!
>
> What factors might affect a species's average lifespan? Make a list on a separate piece of paper.

Maximum life span is the oldest recorded age for a member of a species. Do you know that the oldest recorded human was 122 years old? But humans are not the only ones who can live to be centenarians. A Galapagos tortoise named “Harriet” lived to 175 years old. The Greenland Shark can live longer than 200 years.

It's easy to record the age of an animal in captivity, but to calculate the lifespan of wild animals is trickier. Scientists will often tag animals in order to track and study them. They can then keep a record of a wild animal's age. This is important to measure because animals in captivity generally live longer than wild animals.

> Think about it!
>
> Why do animals live longer in captivity? Make a list of factors on a separate piece of paper.

Scientists can also use physiological clues to calculate a wild animal's age, such as looking at a fish's scales, a turtle's shell, or a whale's earplug. Learn more about these techniques at: http://tpwd.texas.gov/publications/nonpwdpubs/young_naturalist/animals/animal_life_spans/ Note: If you follow the link and read the article, don't look at the table at the bottom of the page as those numbers are used in the next exercise.

Play Along:

Maximum life expectancy can vary greatly between different species. Write the names of the animals in the chart below on index cards or small pieces of paper. Guess the life span of each species and place the cards in the order of your guesses. Next, use the clues to match each species with its record life span (in captivity) to see if you placed the animals in the correct order.

Fill in "Y" for "yes" and "N" for "no" in the grid below based on the clues in this deductive reasoning exercise. A lion has a maximum life span of 35. This is already filled in for you in the grid:

	3	5	10	10	35	50	56	80	123
Golden Eagle					N				
Horse					N				
Lion	N	N	N	N	**Y**	N	N	N	N
Mole					N				
Alligator					N				
Goat					N				
Box Turtle					N				
Guinea Pig					N				
Rabbit					N				
Indian Elephant					N				

Clues:

1. The goat and rabbit have the same record life span.
2. The mole has the shortest record life span of the animals in the list.
3. The guinea pig's record life span is half of the rabbit's life span.
4. Add the life spans of the goat, guinea pig, and lion to get the life span of the horse.
5. The box turtle's lifespan in more than twice that of the horse.
6. The golden eagle's record life span is more than the alligator's

How did the actual life spans compare to your guesses?

Topic 1.2: Kaibab squirrels and geographic isolation

In Chapter 1, Elizabeth spoted something scampering up the tree. “Was it a cat I saw?” she asked. (This sentence is a palindrome!). It turned out to be a squirrel – a Kaibab Squirrel, to be precise. The odd thing about this, as Elizabeth pointed out, is that Kaibab Squirrels are only found on the North Rim of the Grand Canyon (and not anywhere else in the world). Here is a chart comparing the Abert's Squirrel and the Kaibab Squirrel:

	Coloration	Size	Habitat	Food
Abert's Squirrel	Light gray coat, dark stripe down back, white underneath.	Head + body: 18-23” Tail: 7-10”	Ponderosa pine forest in parts of Arizona, Wyoming, Utah, New Mexico, Colorado. Southern rim of the Grand Canyon.	Seeds, bark, buds, and flowers of ponderosa pines.
Kaibab Squirrel	Dark gray sides and underneath, white tails, chestnut brown head and back.	Head + body: 18-23” Tail: 7-10”	Only in the ponderosa pine forest in the northern Kaibab Plateau bordering the Grand Canyon	Seeds, bark, buds, and flowers of ponderosa pines.

Think about it!

Using the table above, identify the similarities and differences between Abert's squirrels and Kaibab squirrels.

In your opinion, do you think these two squirrels are more different or alike?

Think about it!

Can you tell which of the images above is a Kaibab? Which one is an Abert's squirrel?

Scientists previously classified the Kaibab squirrel as a separate species from the Abert's squirrel, but recent genetic research has determined that the Kaibab is actually a subspecies of the Abert's. That means that these two types of squirrels are not only related to each other, but they could potentially mate and have healthy babies. They don't mate, however, because they are separated by the Grand Canyon. Scientists call this "geographic isolation". So how did Kaibab squirrels evolve as a subspecies?

To first understand evolution by natural selection, watch this video from the Howard Hughes Medical Institute. The video introduces you to various species of anole lizards and how their traits help them survive in different habitats.

"The Phylogenetic Tree of Anole Lizards — HHMI BioInteractive Video"
(https://www.youtube.com/watch?v=rdZOwyDbyL0)

Now look again at the Kaibab and Abert's squirrels. They both live in ponderosa pine forests (although in geographically separate regions) and have the same diets. In the case of the Kaibab squirrels, having a different coloration does not provide them with any special advantage specific to the Northern Rim of the Grand Canyon. Instead of evolution by natural selection, Kaibab squirrels are an example of evolution by geographic isolation.

Abert's squirrels formerly occupied both rims of the Grand Canyon and could move freely between the two. About 10,000 years ago, at the end of the last ice age, the rising temperatures cut off access for the squirrels who lived in that ponderosa pine forests on the North Rim of the canyon. The ponderosa trees only thrived in the higher elevations, which were cooler (the North Rim is 1,000 feet higher than the South Rim). The landscape also changed, making climbing down the canyon and crossing the Colorado river impossible for the squirrels. They were trapped! The smaller population of the isolated squirrels bred with each other. Their smaller numbers allowed mutations, such as dark bellies to dominate.

For a basic understanding of genes and heredity, watch this cute video about Mendel's Peas and genetics:

[How Mendel's pea plants helped us understand genetics](https://www.youtube.com/watch?v=Mehz7tCxjSE)
(https://www.youtube.com/watch?v=Mehz7tCxjSE)

Play Along:

Let's use a simplified example to explore how Kaibab squirrels' different coloration might have evolved. The Abert's squirrels all have white bellies. 10,000 years ago, the squirrels who were isolated in the North Rim also had white bellies. Now let's say that one of these squirrels had a genetic mutation that resulted in a dark belly. When the dark-bellied squirrel mated with a white-bellied squirrel, some of their babies may have had dark bellies. Those children would then mate and produce more dark-bellied squirrels. Now the population has squirrels with both dark and white bellies. There is no distinct advantage to belly color in squirrels, so how does the allele, or gene variation, for white bellies completely disappear in the Kaibab squirrel population? Scientists believe that this happened through a process called genetic drift. Try modeling the phenomenon as follows:

1. Gather objects of two different colors (2 colors of marbles/blocks, 2 different types of nuts/candy, etc..) and a number of small bags or bowls

2. Place five objects of each color in a bowl. This is your starting population, with 10 members.

3. Randomly pick five objects from your bowl to represent the members of your population that survive and bread.

4. Let's pretend that each individual has one child (offspring) of the same color.

(You know from the video on Mendel's peas that this is not exactly how inheritance works, but we're just creating a simple model). So place each object you picked along with another object of the same color into the next bowl. For example, if you picked 2 black blocks and 3 white blocks, put 4 black blocks and 6 white blocks in the bowl. This is the second population. (We're keeping the population size the same for the model).

5. Repeat the process - randomly pick five objects from this new population and then place them, along with their offspring into a new bowl. Keep going until your population consists of only a single color.

Record your results:

Population	Number of 1st color	Number of 2nd color
1	5	5

Think about it!

How many generations did it take before all the members were the same color?

How do you think the number of generations is affected by the size of the population?

If you'd like, try the experiment again starting with 10 objects of each color to represent a larger population (or 3 of each color to represent a smaller population).

You just modeled the process of genetic drift, where an allele disappears completely from a population by random chance. This is called fixation. Since the population of Kaibab squirrels living on the North Rim of the Grand Canyon is small and completely isolated, or separated, from other squirrels, this type of genetic drift occurred over the last 10,000 years until all members had dark bellies and the allele for white bellies was completely eliminated.

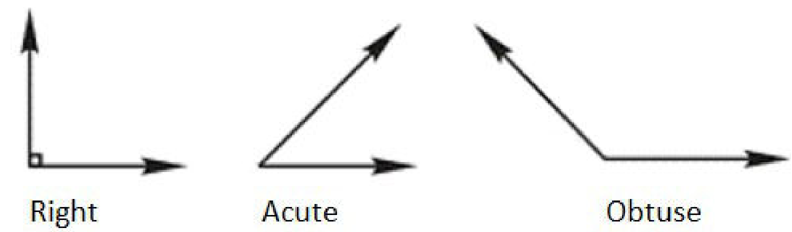
Even though the Kaibab squirrel is considered a subspecies of the Abert's squirrel, they remain separated by the Grand Canyon. It is likely that the Kaibabs will continue evolving and will one day be genetically different enough from the Albert's to be considered their own species.

Bonus Activity: In Chapter 1, Elizabeth stated that magpies do not fly on silent wings, put owls do. Learn more about the science of silent flight here: http://www.bbc.co.uk/nature/16593259

Chapter 2: Magic Square

Topic 2.1: Angles in nature

In the second chapter of "*Math and Magic in Wonderland*", the girls had to classify and count the different types of angles in a Tangram set (with the aid of a pun-loving Kaibab squirrel). Let's explore these different angles and where they are found in nature.



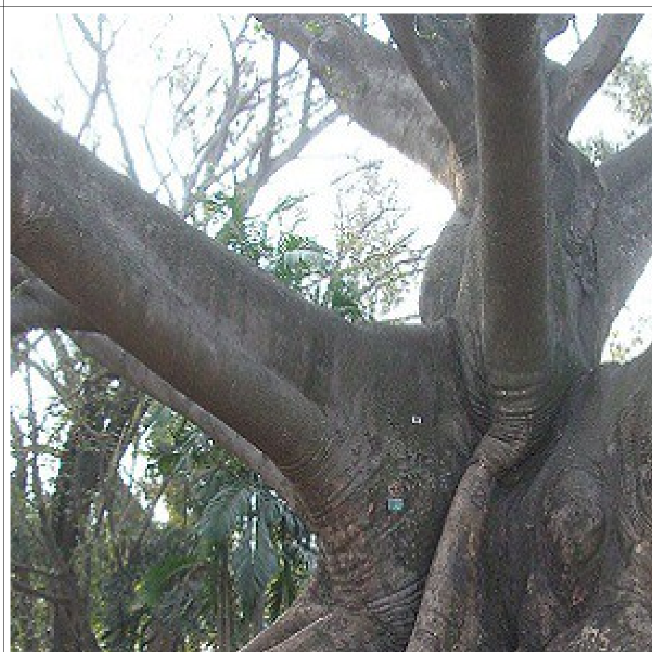
1. A right angle resembles the corner of a book, where two perpendicular lines meet. It is also called a 90-degree angle.

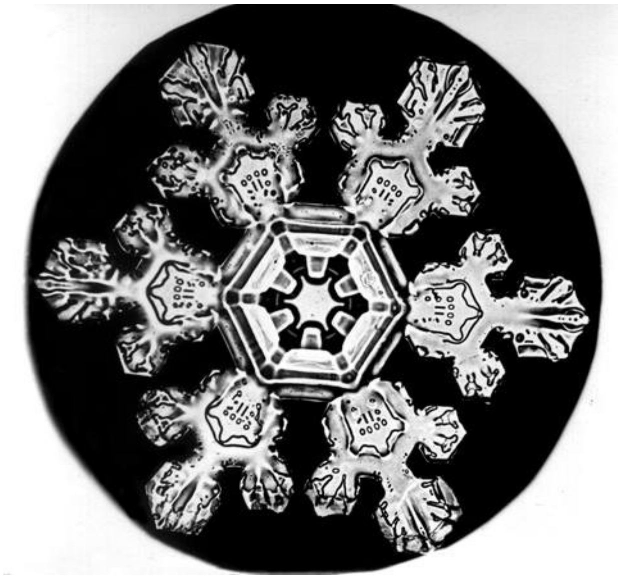
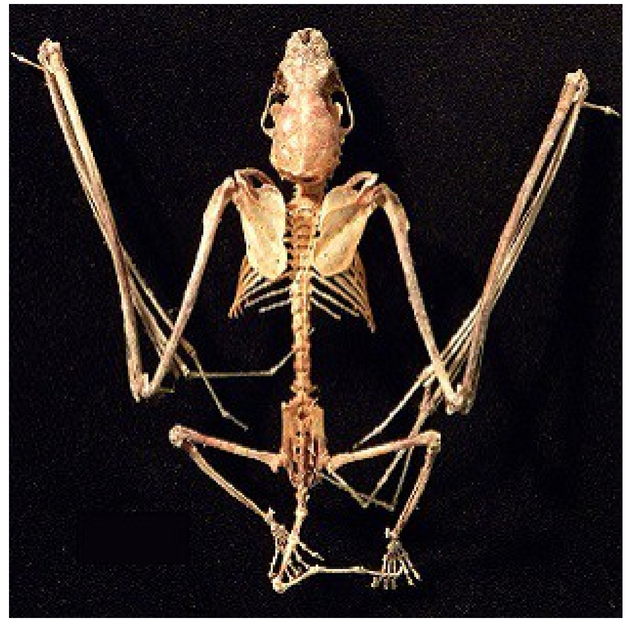
2. An acute angle is "more closed" than a right angle. You can remember it as a "cute" (small) angle.

3. An obtuse angle is "more open" than a right angle.

Play Along:

Can you identify the types of angles in these images?

 Fly Agaric Mushroom	 Cockchafer (Maybug)
 Pyrite ("Fool's Gold") Crystal	 Kapok Tree

 Snowflake	 Bat Spine and Ribs

<u>Nature Walk</u>

Go on a nature walk and identify the angles you find as right, acute, and obtuse. Record your findings in a journal.

The riddle in Chapter 2 of Math and Magic in Wonderland asked the girls to “*write how many fewer rights there are than all the others*”. How did the number of right angles you found on your walk compare to the number of acute and obtuse angles? Why?

Were you intrigued by the right angles on the Pyrite crystal? What three dimension shape do you see in the Pyrite? Let's dig deeper (without writing an entire chemistry or physics course!)...

<u>Think about it!</u>

The Pyrite crystals form in the shape of a cube**,** a three-dimensional shape with six square faces. Since this chapter is about angles, calculate the number of right angles in a cube.

Many crystals form in cubic shapes. Use a magnifying glass or microscope to examine table salt. What shape are the crystals?

Let's explore how crystals like Pyrite and salt get their shape. All chemical elements (solid, liquid, and gas) are made up of units called atoms which contain positively-charged protons and negatively-charged electrons (think of the positive and negative poles of a magnet). When these atoms get charged by gaining or losing one or more electrons, they are called ions. Charged atoms, or ions, stick together by electrostatic attraction. "Electrostatic" might sound like a big word, but you've probably heard of "static electricity", which is the same concept.

Play Along:

Rub a balloon against your shirt until it collects enough extra electrons (a negative charge) for the balloon to stick to a wall. (If you don't have a balloon handy, you can try this virtual lab: https://phet.colorado.edu/en/simulation/balloons).

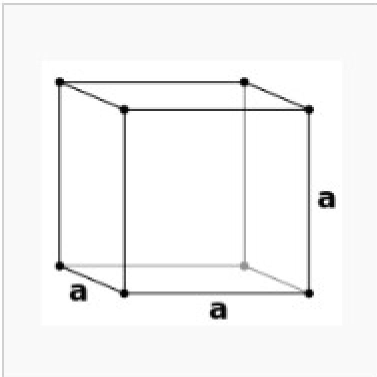
This video does a great job explaining static electricity:

Bill Nye The Science Guy on Static Electricity
(https://www.youtube.com/watch?v=Z-77IzaXGcg)

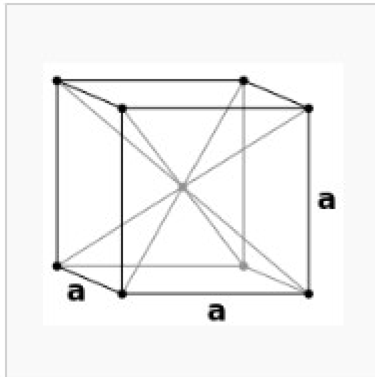
The ions inside solid crystals bond (stick together) using the same mechanism that made your balloon stick to the wall, but on a much much smaller level, which you can't see with the naked eye. The ions arrange themselves to form a symmetrical three-dimensional structure called a crystal lattice, which often resembles a cube. If many of these structures pile on top of each other, you get a crystal that can be seen with your eyes.

Play Along:

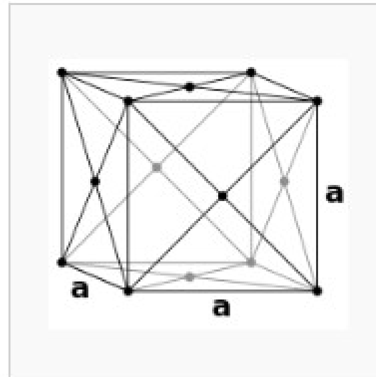
Using marshmallows and toothpicks, see if you can build these cubic crystal structures:



Simple cubic (P)



Body-centered cubic (I)



Face-centered cubic (F)

Think about it!

You already determined that a simple cube has 24 right angles. Use your marshmallow crystal models to count the number of each type of angle in the body-centered cubic and face-centered cubic structures.

Bonus Activity: Purchase a crystal growing kit, grow borax crystals (instructions here: http://www.danslelakehouse.com/2015/01/diy-borax-crystals.html), or grow crystals on a string using sugar, table salt, or Epsom salts (instructions here: http://www.sciencecompany.com/Simple-Crystals-on-a-String.aspx)

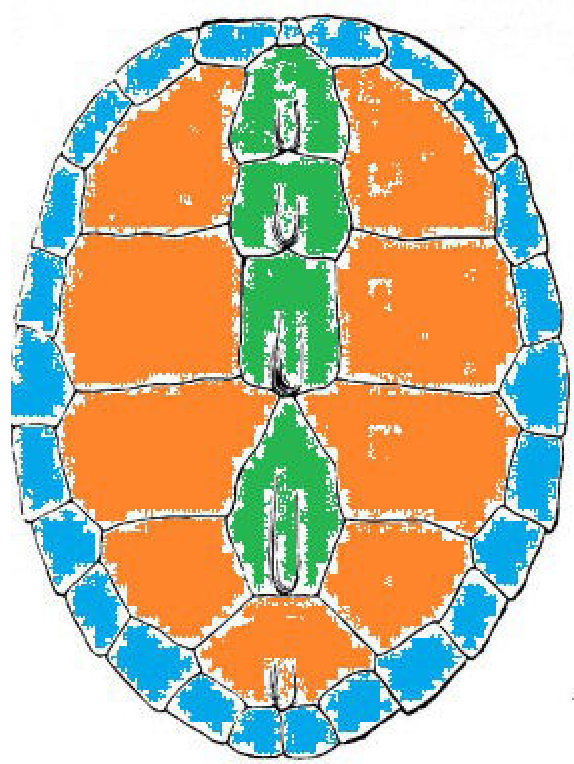
Topic 2.2: Turtle's carapace

In Chapter 2 of "*Math and Magic in Wonderland*", Lulu and Elizabeth formed the shape of a turtle using Tangrams and solved a magic square on the turtle's back. Elizabeth correctly pointed out that a turtle's back is called a carapace (while the underside part of its shell is called a plastron.)

A turtle's carapace consists of bone, covered with a thin layer of skin, followed by a layer of hard protective scutes which are made of keratin, the same material that your fingernails are made of. All turtle species have this same shell structure. Turtles typically don't carry magic squares on their backs, but there *is* something magically mathematical about the number and position of the scutes on their carapace.

Play Along:

Using the diagram of a turtle's carapace, count the number of scutes in each section:

Scute Count	
1st section (green): _____________ 2nd section (orange): _____________ 3rd section (blue): _____________	

Do you notice anything special about the number of scutes in each section? Gather some square building blocks (or another collection of small objects like crackers). Use the objects to try to arrange the number of scutes in each section into a square (with the same number of objects in each row and column). These are called square numbers. I suppose turtles have magic tricks up their sleeves- I mean backs - after all! Square numbers will come up again in Chapter 10 of the novel.

Bonus Activity:

In the novel, Elizabeth identified the Tangram turtle as a "sea turtle" because of its flippers. Explore the differences between turtles, terrapins, and tortoises using these resources:

http://www.diffen.com/difference/Tortoise_vs_Turtle

http://animals.sandiegozoo.org/animals/turtle-tortoise

http://www.wikihow.com/Tell-the-Difference-Between-a-Tortoise,-Terrapin-and-Turtle

Chapter 3: Secret Codes

Topic 3.1: Of cabbages and spirals

In Chapter 3 of Math and Magic in Wonderland, the twins deciphered a secret message that told them to bring a cabbage in their baggage! Mrs. Magpie obviously loves riddles, but nature, too, is full of secrets. If Lulu and Elizabeth had cut open their cabbage, for example, they would have found another "secret code":

Nature Walk

Can you see the spiral inside the cabbage? Go on a nature walk and try to find other examples of spirals in nature.

Play Along:

Most spirals in nature, including pine-cones, sunflowers, and snail shells, have a hidden mathematical pattern. Find the pattern in the following sequence and then calculate the next three numbers. (Hint: To find the pattern, add 1+1, 1+2, 2+3. Do you see it?)

1, 1, 2, 3, 5, 8, 13, 21, _____, ______, ______

This pattern is called the Fibonacci sequence. It might come in useful in later chapters of "*Math and Magic in Wonderland*" (hint, hint!).

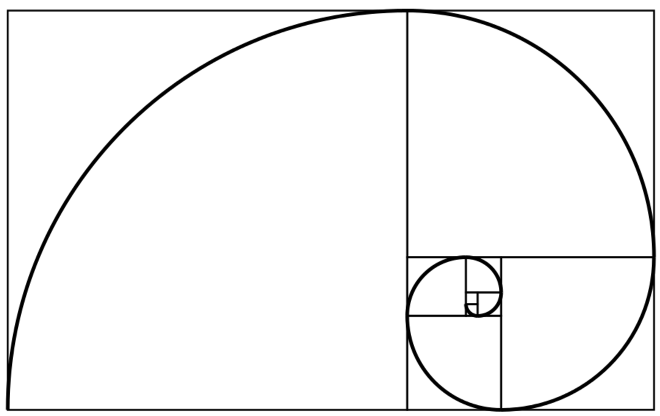
Play Along:

Here's how the sequence relates to spirals:

1. Using a ruler, draw a square that is 1 cm by 1 cm (use the corner of your ruler to make sure that you have 4 right angles in your square). Cut out two of these squares.

2. Next, draw and cut out a square that is 2 cm x 2 cm, one that is 3x3, others that are 5x5, 8x8, and so on (where the length of the sides of each square are Fibonacci numbers).

3. Place (don't glue or tape) the pieces of paper in this formation, and draw a spiral through them:



You used Fibonacci numbers to create a special spiral, which is also called a golden spiral. The cabbage might not look exactly like this, but the growth pattern of its leaves followed this pattern before they were compressed into the cabbage's round shape. Did you ever notice that the leaves on the outside of a cabbage are the largest, and the leaves get subsequently smaller as you move toward the center? Nature has a reason for its secret patterns. Cabbage leaves are tightly packed, and the Fibonacci sequence and golden spiral pattern allow cabbages to make the best use of space.

Play Along:

Take the square pieces of paper that you used for creating your spiral and tape them together (in order) in <u>one straight line</u>. Only place tape on one side of the paper so there are joins allowing some flexibility between the squares. Starting with the end of the paper that contains the smallest square, roll your paper into a “cabbage”. Did you notice how the Fibonacci sequence helped?

Topic 3.2: Cryptography with Biology

How can humans use natural processes to send secret messages? Scientists often get ideas by observing the world around them.

Play Along:

1. Take a fruit (an apple, banana, pear, or avocado will work) and slice it into smaller pieces.
2. Leave the pieces of fruit out on the counter, where they are exposed to the air.
3. Check back after an hour and make observations.

Did you notice a change in your fruit's color? This is a result of a process called oxidation. You can learn more about the chemistry behind oxidation here:

http://kitchenscience.sci-toys.com/oxidation

Play Along:

Do you think that the process of oxidation can be used to send secret messages? Try making your own 'invisible' ink:

1. With adult supervision, cut a lemon in half.
2. Squeeze the lemon juice from one of the halves into a bowl. (Save the other half for making lemonade!)
3. Add ½ teaspoon of water to the bowl and mix it with a spoon.
4. Get a sheet of white paper.
5. Dip a clean paintbrush or Q-Tip into your lemon juice and use it to write a message on your white paper.
6. Let the paper dry. The message should be “invisible”.
7. To reveal your message, hold the paper next to a heat-emitting lightbulb or other source of heat (like a space heater or heat-pad). Be careful not to burn yourself or do anything that could start a fire.

As the lemon juice oxidized, your message became visible. The job of the heat was to speed up the process, but it would have occurred eventually if you had left your paper out. Messages written in lemon juice would not be the best for spies to carry for long periods of time.

Scientists have recently started to explore ways of sending information using living organisms (and not just carrier pigeons!). In this new field of study, called InfoBiology, messages are typically written in bacteria (microscopic organisms that sometimes make you sick).

Like all living things, bacteria has DNA which carries genetic information (remember Mendel's peas and the Kaibab squirrels?). Bacteria is very simple, and scientists not only understand its genetic code, but they can also modify its genes to give bacteria different traits. For example, they can make changes to bacteria genes to give the bacteria different colors. This is called fluorescence. Just like your "invisible ink" was activated (revealed) using heat, the bacteria's color can be unlocked by putting it in a solution of lactose (a sugar) and a specific type of antibiotic. Antibiotics usually kill bacteria, but bacteria can have genes that cause them to be resistant to a certain antibiotic (so they don't die when exposed it).

Side note: Antibiotic-resistant bacteria is really dangerous if it is disease-causing because they will not respond to treatment using the type of antibiotic to which they are resistant. Fortunately, scientists performing InfoBiology experiments use bacteria that is generally considered safe.

In 2011, a team of scientists (Palacios, M. A. et al.) placed antibiotic resistant genes inside bacteria and linked them to fluorescent proteins of a certain color. The scientists used the bacteria to "write" a secret message, whose colors don't appear until they are activated. When the bacteria is put in a solution containing lactose and the right type of antibiotic, it glows in the color the scientists had intended. The color of the bacteria holds an encoded message. Just like Lulu and Elizabeth had to decode Mrs. Magpie's clue by matching numbers to letters in the alphabet, the colors of the bacteria must also be matched to letters in the alphabet.

Think about it!

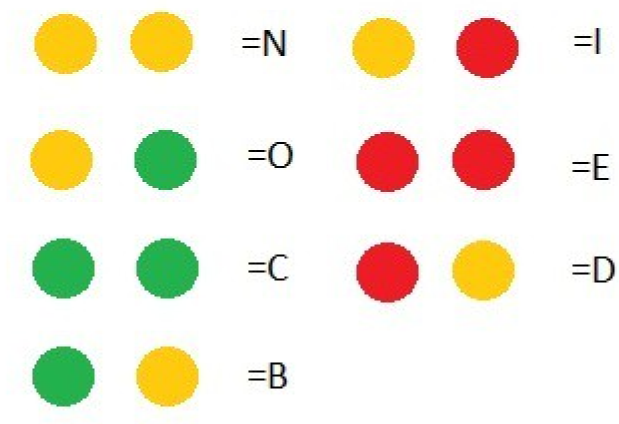
The scientists who developed this bioencoding method decided to use a pair of colors to stand for each letter. For example, a well of yellow bacteria, followed by a green well would indicate the letter "H". Why do you think they had to use two colors to stand for a letter instead of only one? (Hint: How many letters are in the English alphabet? How many colors can you name?)

Chapter 3 of "Math and Magic in Wonderland" also introduced you to permutations**.** Be sure to complete the "Play Along" section at the end of the chapter, if you haven't done so already.

Think about it!

If there are 26 letters in the alphabet, and every pair of colors stands for a letter, what is the minimum number of different colors you would need (to be able to use any letter in the alphabet for your message)? (Hint: If you used three colors, you would have three choices for the first color and three choices for the second color, or 3 x 3 = 9 permutations. You need more permutations than that to allow for 26 different letters).

Here is a key matching pairs of colors to letters:



Use the key to decode the following word (the first letter is done for you):

Chapter 4: Rabbit Trails

Topic 4.1: Circles in nature

In Chapter 4 of "*Math and Magic in Wonderland*", Mrs. Magpie challenged Lulu and Elizabeth to draw a "perfect circle portal", which they finally accomplished with the aid of a homemade compass.

Nature Walk

Giotto's circle-drawing ability won him a job painting frescoes at the Basilica of St. Francis. Surely circles are not just the creation of an artist's fancy. Go on a nature walk and see how many circles and spheres you can find.

Think about it!

Are any of the circles you found on your walk "perfect circles"? That may be a question for a philosopher. Greek philosopher Plato (427- 347 BC) thought that a perfect circle is an "ideal form" which isn't actually found in nature (so a flower may resemble a circle, but it is never perfect). What do you think?

Lulu and Elizabeth's method of drawing circles (with a compass) reveals one of the attributes that make circles a "perfect" shape – rotation symmetry of an infinite order. If a circle is turned (rotated) around its center, it will always look the same. A circular man-hole cover will always fit its man-hole regardless of which way you turn it (and will never fall in). In three dimensions, objects which look the same if they are rotated on one axis have cylindrical symmetry (they don't have to be cylinders to have this). Just imagine poking a pin through the object (this is the axis) and spinning it around. If the object looks the same any way you spin it, it has cylindrical symmetry. With a perfect sphere, you can stick your pin (axis) anywhere. This is called spherical symmetry.

Play Along:

Many pearls have rotational symmetry. Use modeling clay or dough to form some of the three-dimensional objects found in the pearls in this diagram. Pay attention to the technique you use to make the shapes. Try to find the axis of rotation in each of your clay shapes and use a pencil or pin to test it's rotational symmetry.

Think about it!

Which of these pearls have cylindrical symmetry?
Which have spherical symmetry?

Scientists have found an explanation for the rotational symmetry in many pearls. Just as you rolled the modeling clay to form your pearl models (ensuring that they are the same shape around your axis of rotation), real pearls rotate inside their mollusk-shell homes. You can read more about how pearls get their shape here:

http://www.bbc.com/future/story/20130623-how-pearls-get-their-round-shape

Let's explore another example of spheres in nature: bubbles.

A bubble is a sphere of gas (like air) in a liquid (like water):

Siamese Fighting Fish encase their eggs in protective bubbles.

Play Along:

Are all bubbles spherical or can they form different shapes?

1. Gently mix the following to create bubble solution:

    -6 cups water
    -1 cup liquid dish soap
    -1/4 cup corn syrup

2. Bend pipe cleaners into different shapes to make bubble wands.

3. Dip into your bubble solution and blow.

> Think about it!
>
> What shape were your bubbles?
> Did wands of different shapes produce different-shaped bubbles?

This experiment revealed another aspect of a circle's perfection - its efficiency. If a farmer had a certain length of fencing and wanted to fence off the largest field he could, he would put the fence up in the shape of a circle because it has the largest area for a given perimeter. Similarly (in the 3-D world) the surface tension (the force that pulls the water molecules together) packs the soap water particles into the tightest bundle possible. The air molecules inside the bubble push outward on the soap film with the same force in every direction and balance with the surface tension pushing the bubble inward. The result is that a bubble always has the smallest surface area for the volume of air it contains. The most efficient shape for accomplishing this is a sphere.

> Think about it!
>
> If bubbles are always spherical, why can balloons, which are also filled with air, hold a variety of shapes? (Buy different-shaped balloons – long and narrow, heart-shaped, etc.. - to experiment)

Topic 4.2: Rabbit trails of famous scientists

"Galileo, Newton, Descartes, and Pascal too,
All followed rabbit trails that led to something new."

In the "Play Along" section of Chapter 4, you matched these famous mathematicians with the rabbit trails they followed. Scientific rabbit trails often lead to new discoveries and can even have an impact on human interaction with the natural world. For example, when marine biologist Rachel Carson (1907-1964) returned to her hometown in rural Pennsylvania, she noticed that there seemed to be fewer birds around compared to the numbers she remembered as a child. This observation led her down a rabbit trail which linked the decline in bird populations to pesticides that were being sprayed on farms in the area. Carson began to research the effects of pesticides on both animals and humans, which led her to make important connections between pesticides and cancer. Her book, "*Silent Spring*" informed Americans of the harmful effects of pesticides and played an important part in environmental protection and conservation.

Rachel Carson

Nature Walk

Follow your own scientific "rabbit trails". Take a notebook on a nature walk and make a list of questions about your observations. When you get home, pick a question or two to explore in more detail using a computer search engine, encyclopedias, or books from your local library.

One question often leads to another, so keep digging to see where the rabbit trail leads you. Make sure that you never dismiss your questions as silly – some of the world's greatest discoveries have come from people who pursued things that others thought were impossible.

"Alice laughed: 'There's no use trying,' she said; 'one can't believe impossible things.' 'I daresay you haven't had much practice,' said the Queen. 'When I was younger, I always did it for half an hour a day. Why, sometimes I've believed as many as six impossible things before breakfast.'"

- Lewis Carroll, "Alice in Wonderland"

Chapter 5: Two Worlds Join

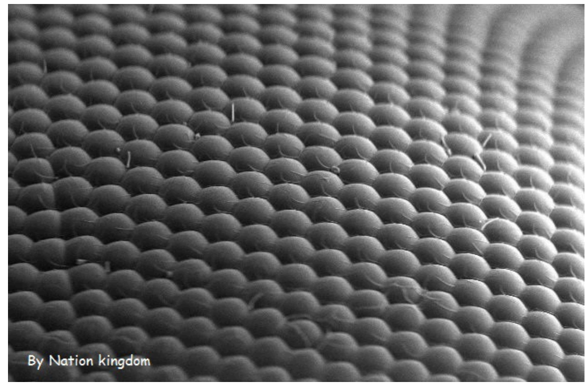
Topic 5.1: Tessellations in nature

In Chapter 5 of "*Math and Magic in Wonderland*", Lulu and Elizabeth finally enter Mrs. Magpie's world – Wonderland! Being excellent observers, they immediately notice some peculiar things about this new world. One of these peculiarities is that the leaves can be tessellated. They can be placed to cover an area without any gaps or overlaps. While most leaves here on Earth do not form perfect tessellations, looking closely at the leaf itself will reveal...

… tessellations!

In the nature activities for the last chapter, we learned that circles are efficient shapes (they contain the greatest area for a given perimeter). Nature is always trying to be efficient, and tessellations are a great way to fill up a space without any waste (gaps). Here are some examples of where tessellations can be found in nature:

Giraffe Spots



House Fly's Compound Eye
(under electron microscope)

Fish Scales

Pineapple Skin

<u>Nature Walk</u>

Go on a nature walk and find more tessellations. Make sketches in your nature journal.

A classic example of a tessellation that uses regular polygons is a bee's honeycomb:

Can you see the eggs and larvae in this honeycomb?

<u>Think about it!</u>

Which polygon is being tessellated in the honeycomb?

<u>Play Along:</u>

If a sphere is such an efficient shape, why don't the bees use spheres instead? Try this:

1. Make some bubble solution using the recipe found in the science activities for the last chapter.

2. Pour a bit of the bubble solution onto a flat dish.

3. Have a friend (or parent) hold a flat, clear glass plate or lid a couple inches above your dish. You could use a clear pot lid, Tupperware lid, or even a glass casserole dish.

4. Use a drinking straw to blow bubbles blowing bubbles into your dish and observe their shapes from the top lid.

<u>Think about it!</u>

What shape did the bubbles form? Why?

You should have seen the bubbles take on a tessellating pattern similar to that of a honeycomb. Multiple spheres are not the best shape for filling up a space. Remember that nature has efficient arrangements, so the bubbles formed into shapes that tile without wasted space – tessellations!

Bonus Activity:

To see how inefficient spheres can be at filling a space, try this:

1. Fill a jar with gumballs or marbles

2. Pour sand into the jar and observe all the gaps the spheres left (which are now filled up by the sand)

3. Look at the particles of sand with a magnifying glass. Are these shapes efficient at filling space?

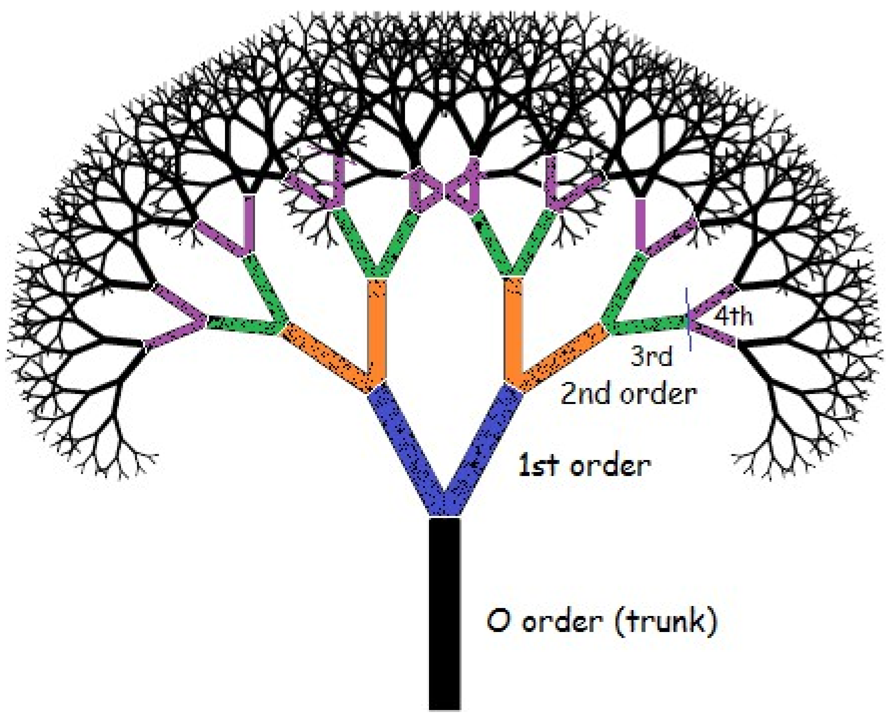
4. Pour water into the jar and observe all the gaps left by the sand.

Topic 5.2: Fractals in nature

Another observation that the twins made in Chapter 5, was that the trees followed a fractal branching pattern, where each part of the tree (all the way from the trunk up to the smallest twigs) had the same characteristics as the whole tree. Fractal patterns are not limited to trees in Wonderland, however. Trees in our world are examples of fractals, as well, although they don't follow the exact pattern described in the novel (where each branch slits into two branches).

Play Along:

Let's take a closer look at tree branching patterns:



We'll label the tree trunk as the zero-order branch, the next level of branches as first-order branches, and so on. Record how many branches there are in each level for the fractal tree found in Wonderland (the computer generated model above):

Branch Order	Number of Branches
0 (black)	1
1 (blue)	2
2 (orange)	
3 (green)	
4 (purple)	

Think about it!

Do you notice a pattern? Can you predict how may branches there will be in the 5th branch order? (Note: this pattern may come in handy later in a future chapter of Math and Magic in Wonderland)

Now find a natural tree growing outside and attempt to count the number of branches in each order:

Branch Order	Number of Branches
0	1
1	
2	
3	
4	

Think about it!

How did this tree compare to the computer-generated fractal tree?

Repeat with different species of tree and compare. How can the number of branches at each order can tell you something about the shape of a tree?

The idea for this activity was inspired by the Fractal Foundation's free online fractals course, which you an view here: http://fractalfoundation.org/OFC/OFC-index.htm

Artist and inventor, Leonardo da Vinci (1452-1519) was fascinated by the fractal nature of trees. He wrote in his notebook that if you add up the area of the cross-sections of all the child branches of a tree at any point, the sum would equal the area of the cross section of the mother branch.

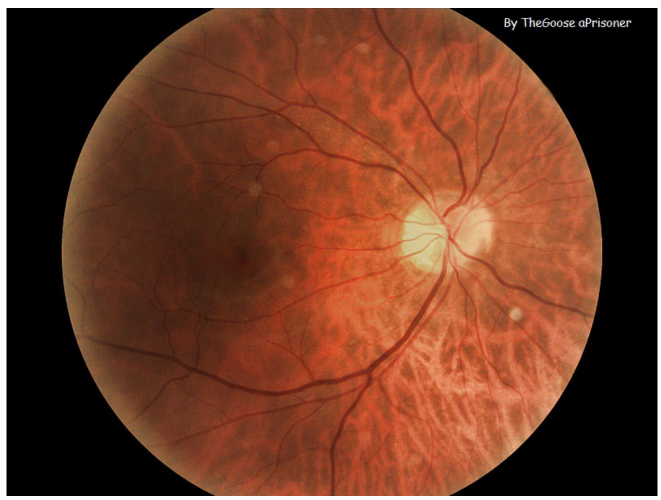
Nature Walk

The next time you find a fallen tree bough, take some measurements of the main branch as well as the child branches and come up with your own theory of how their relative sizes.

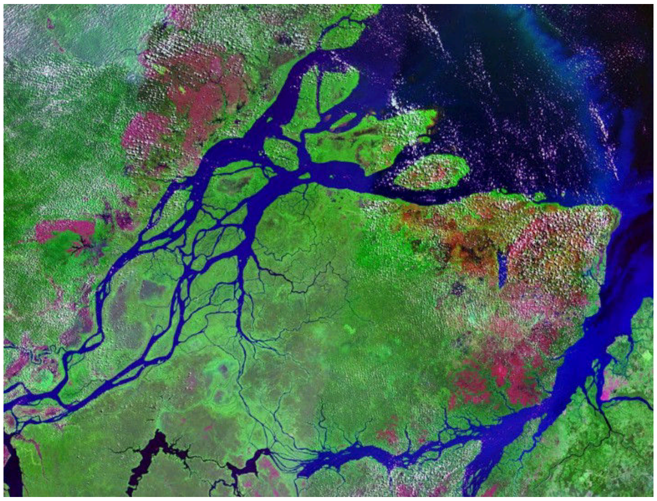
Why do trees (generally) follow da Vinci's rule for branch thickness? French scientist Christophe Eloy generated computer models of fractal trees and found that the ones that followed da Vinci's pattern, were best able to withstand high winds (in a virtual wind tunnel).

Here are some additional examples of branching fractals in nature:

Fractal Lightning



Fractal Vessel in a Human Eye



Fractal River Network (Amazon River)

Fractal Patterns in Frost

Lightning, blood vessels, rivers, and frost follow a similar fractal branching pattern to trees, but they do not have to stand upright in high winds. What other reasons can you come up with for the branching pattern in each of these cases?

Nature Walk

Find more examples of fractals on your nature walk. Remember that fractals do not have to branch; Any pattern is repeated over and over at every scale is considered a fractal. (Spirals also fall into this category.)

Chapter 6: River Crossing

Topic 6.1: Buoyancy and fish

In Chapter 6 of "*Math and Magic in Wonderland"*, the girls had to take a cabbage, a pig, and a wolf across a river aboard a leaky boat. The boat took on more water with each river-crossing trip, but managed to stay afloat. Chapter 6 of the Book Club (at http://learnersinbloom.blogspot.com) invited you to experiment with different boat models, but let's talk about buoyancy (an object's tendency to sink or float) in a general sense.

> Think about it!
>
> If you take a tree branch and a rock both weighing the same and place them in water, what will happen? (Try it!)

Play Along:

The tree branch is positively buoyant (it floats), and the rock is negatively buoyant (it sinks). If they both weigh the same, then weight can not be the only factor in buoyancy. Try this:

1. Fill a small glass cup or bowl with water and mark the level of the water on the outside of the glass using a dry-erase (or other washable) marker.

2. Drop a rock (or other small object that sinks) into your cup and mark the new water level.

Did you observe that the water level rose when an object was placed into the water? The amount of water displacement for an object that sinks is equal to the volume of the object, or how much space it takes up. Even objects that float will displace water, but if an object's density (weight divided by volume) is less than the density of the fluid in which it is placed (water in this case), then the buoyancy (upward force) on the object will be greater than the force of gravity (the downward force), and the object will float. These buoyancy principles were developed by the Greek inventor Archimedes (287 – 212 BC)

Play Along:

Here's another fun experiment to try:

1. Fill a cup or bowl with water. (You can use any container, but it needs to be larger than an orange.)

2. Place an orange into the water. Does it sink or float?

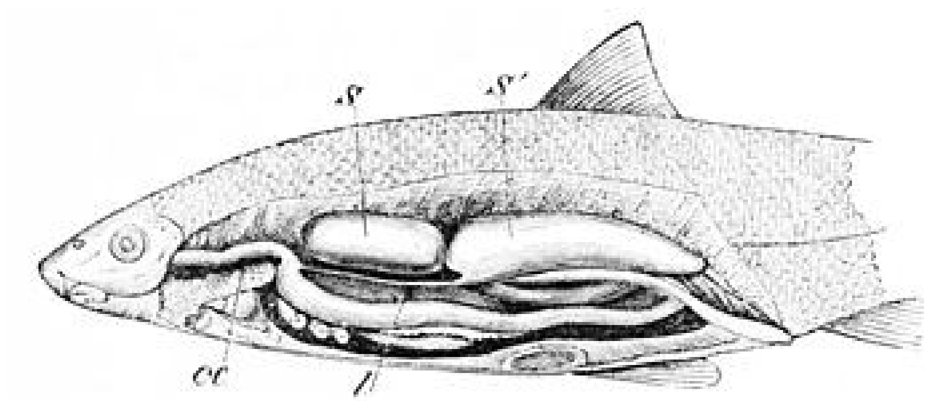
3. Peel the orange and place it back in the water. What happens?

4. Add salt to the water until you notice the orange moving.

> Think about it!
>
> How did the *density* of the orange change when you peeled it? How did the *density* of the water change when you added salt?

The Dead Sea is almost ten times as salty as the ocean. Humans can easily float on its surface, but the Dead Sea is too salty for fish and aquatic plants to survive.

Fish use buoyancy principles to move up and down in the water. Most fish have an organ called a swim bladder (or a gas bladder), which allows them to control their buoyancy in the water. With just the right amount of gas in its swim bladder, a fish can make sure that it has neutral buoyancy, which means that it neither floats nor sinks but can control its vertical (up and down) movements in the water. Some species of fish move the gas in and out of their swim bladders through their guts. (Have you ever seen a fish appearing to gulp air or blowing bubbles?) Others can move oxygen between their bloodstream and swim bladder directly.



The diagram above shows you the position of a fish's swim bladder (labeled "S"). If you dissect a fish, you'd find an organ that looks like this:

Swim bladder!

Play Along:

Learn more about the buoyancy principles of a swim bladder:

1. Fill your bathtub with water.

2. Use an empty plastic water bottle for your "swim bladder" and verify that it floats in the tub (positive buoyancy).

3. Add a small amount of water to your bottle and test to see if it still floats.

4. Keep adding water in small amounts until you've achieved neutral buoyancy (neither floating nor sinking). If you accidentally add too much water, creating negative buoyancy (sinking), simply remove some water.

5. Experiment with adding and removing small amounts of water to show how a fish's swim bladder allows it to move vertically in the water.

Submarines use the same concept as the swim bladder to control their depth in the water. Oceanographers use floats that rise and sink in this way to take measurements at different ocean depths. See a diagram here: http://www.argo.ucsd.edu/floatcyclescaled.jpg

Topic 6.2: Wildebeest migration

The river-crossing problem in Chapter 6 involved talking animals (and a cabbage), but in the natural world, large mammals often have to undertake their own dangerous river crossings.

Every July, enormous groups of wildebeests gather at the edge of the Mara River. Over a million of them will cross the crocodile-infested waters, leaving the drying Serengeti of Tanzania behind as they follow the rains to Kenya's Maasai Mara Game Reserve. Scientists are interested in tracking the migration patterns of wildebeests and estimating their population in order to determine what impact humans are having on these creatures (as they develop areas and build roads that might cut wildebeests off from their traditional migration routes). But traditional tracking methods such as tagging the animals with GPS collars or performing aerial surveys are difficult and not always precise.

Scientists at Dartmouth University have developed a method for identify individual wildebeests in photographs based on markings that a computer program can tell apart. The technology, called "Wild-ID" only has an 8% error rate for wildebeest identification. Scientists have already used this technology to confirm their suspicions that there is a decline in wildebeest migration routes and population compared to historical data previously collected using traditional tracking methods.

<u>Think about it!</u>

1. If the *Wild-ID* software is wrong 8% of the time, what percentage of the time is it correct?

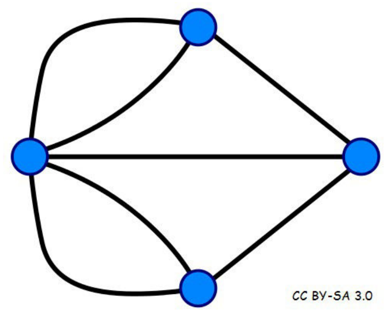
2. If *Wild-ID* has identified 10,000 wildebeests, approximately how many of those identifications will be correct?

3. If the wildebeest herd population was 1,500,000 a decade ago, and is currently 1,200,000, how many fewer wildebeests are there now compared to 10 years ago? Can you express this population decline as a percentage?

Chapter 7: Seven Bridges

Topic 7.1: Graph theory and food webs

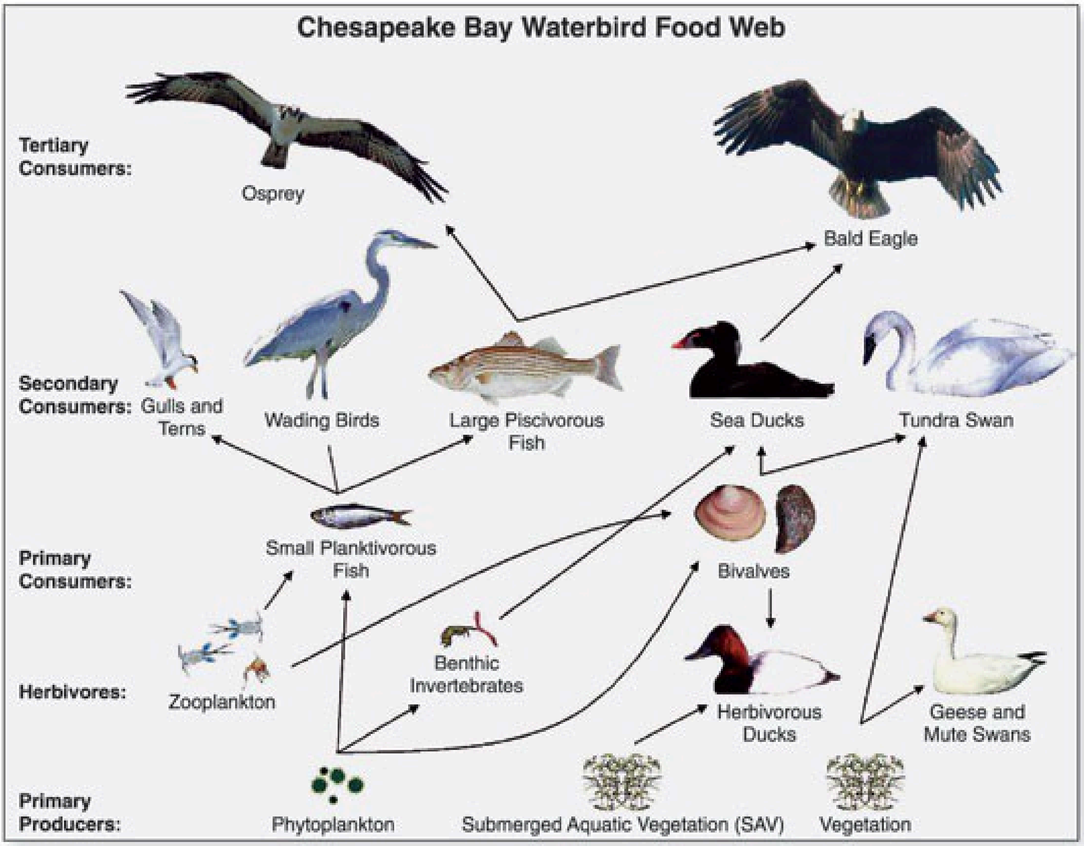
The bridge-crossing map in Chapter 7 of "*Math and Magic in Wonderland*" looked a lot like the classic math problem of *The Seven Bridges of Konigsberg*. In fact, in graph theory, which Euler started with the "Seven Bridges" problem, the two maps are identical (or isomorphic). Their graph would look like this:



In this graph, the nodes (circles) represent the different landmasses (islands and mainland) and the edges (lines) between them show the way those landmasses are connected to each other (bridges). Diagrams such as this simplify the problem and make it easier to come up with rules. For example, Euler labeled a node as either odd or even based on how many edges connected to it. He theorized that an odd node would have to be at either the beginning or end of a path (which passes through each edge only once). This makes the "Seven Bridges" problem (with 4 odd nodes) impossible to solve.

The researchers studying the wildebeest migration routes in the last chapter's activities might make diagrams similar to Euler's. Graph theory, however, has many more applications beyond modeling landmasses and bridges. For example, the interaction between species in an ecosystem can also be modeled in a graph.

Food webs provide a classic example of graph theory. In a food web, the nodes are the species and the edges represent the energy transfer (when one species eats another). In a food web, the arrows point from prey to predator because the prey's energy is transferred to the predator when it is eaten. The diagram shows direct relationships (edges, or arrows from one species to another), but it also shows indirect relationships (paths through the web by following edges). Modeling these relationships help scientists understand how species in an ecosystem are interconnected and assist them in predicting what would happen if a certain species's population increases or decreases.



By Matthew C. Perry

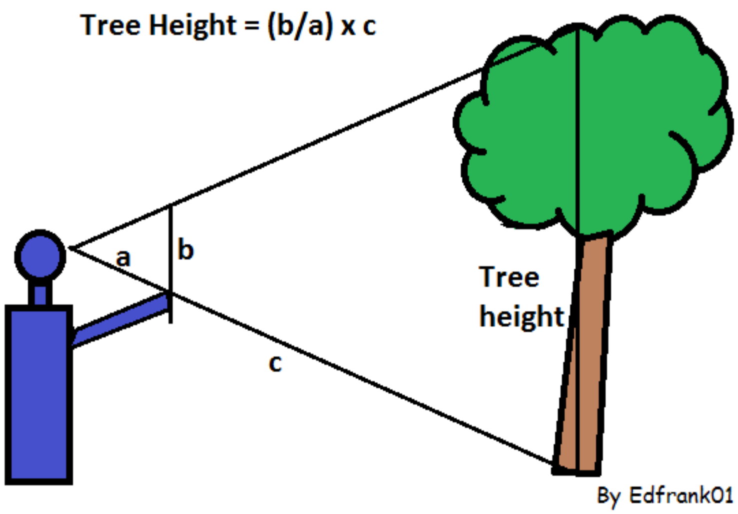
<u>Think about it!</u>

Answer these questions about the food web above:

1. What do the primary producers have in common? Where do they get their energy?
2. What does the Bald Eagle eat?
3. Who consumes the bivalves?
4. Name all the species which consume both plants and animals?
5. Filmmakers recently caught zooplankton eating plastic (see the video here: https://www.nrdc.org/onearth/palate-plastic). When zooplankton consume microplastic (tiny pieces of plastic), they may absorb the plastic chemicals into their bodies. Which other species in the food web would also be affected?

Topic 7.2: Measuring the height of a tree

In Chapter 7, the girls used Thales's method to determine the height of a tree using the proportionality of shadows. In a forest, however, a scientist would not be able to accurately measure a tree's shadow. Instead, scientists will often employ the “stick method”.



Play Along:

Have you ever noticed that if you stand away from a person and close one eye, you can fit the image of that person in your field of vision between your thumb and index finger? (If you haven't, try it now!) The “stick method” of measuring tree height is based on this same concept. Here's how its done:

1. Hold a stick (or pencil) at arms length in front of your face.

2. Stand so you can see the top of the tree at the top of your stick and the base of the tree at the top of the hand holding the stick. You might have to move forward and backwards until your field of vision lines up.

3. Ask a friend to measure the following distances (using a measuring tape):

   A: The distance from your eye to the top of the hand holding the stick.
   B: The distance from the top of the hand holding the stick to the top of the stick.
   C: The distance from your eye to the base of the tree.

4. To find the height of the tree, divide B by A and multiply by C.

How does this method work? Take a look at the diagram. Do you see two triangles? (One has A and B as two of its sides, and the other has a side with length C and a side with the height of the tree). The triangles are similar, which means that their angles are congruent (the same). Similar triangles have proportional sides. Remember Thales's shadow method used in the book? It uses proportionality too! Basically, the ratio between A and C, is the same as the ratio between B and the height of the tree (let's call it "T"). A ratio between two values just means that you divide them by each other. So...

B divided by A = T divided by C

If you know algebra, you can multiply both sides of the equation by C to get T on its own. You can also draw overlapping similar triangles (such as those in the diagram) and measure the length of the sides to verify that this formula works.

<u>Nature Walk</u>

On your nature walk, measure the height of a tree using both the stick method and the shadow method and compare the results. Which tree-measuring method do you prefer?

Chapter 8: Veracity

Topic 8.1: Mimicry

In "*Math and Magic in Wonderland"*, the Mome Raths and Slithy Toves look identical but have different moral codes: Mome Raths always tell the truth and Slithy Toves always lie. The only way of telling them apart is by asking them questions.

In nature, different species may look very similar. Since scientists can't ask organisms questions, they have to use DNA analysis to determine if they belong to different species.

Play Along:

Can you find the six matching pairs of tree frogs in this image?

Now find the eight matching pairs of Heligonius butterflies in this image? Use a stopwatch to record how long the matching task takes you.

How long did it take you to match up the pairs of butterflies? Do you think a predator would be tricked by the similarities in coloring? Well, that is one reason for mimicry (when one species looks like another).

In Batesian mimicry (named after English naturalist Henry Bates), a species that is palatable (tasty) looks like another species that is unpalatable (bad tasting) or poisonous. Predators, who don't have the time to distinguish between the two species or risk making the wrong choice, leave both of them alone, thus helping the mimicking species.

Another type of mimicry is Mullerian mimicry (named after German naturalist Fritz Muller) where both look-alike species are unpalatable. In this case, the mimicry benefits both species since a predator who tastes either one of them will leave the other alone.

Scientists previously believed that the Viceroy butterfly's resemblance to the Monarch was an example of Batesian mimicry. A study in 1991, however, proved that the Viceroy was unpalatable as well, making the relationship an example of Mullerian mimicry.

Think about it!

Look at the images above. The Monarch is pictured on the left, and the Viceroy on the right. The Viceroy is slightly smaller than the Monarch. How else can you tell the two apart? Use your findings to determine which of the butterflies pictured below is the Viceroy and which is the Monarch.

Topic 8.1: Mimic ratio

The ratio between populations of species involved in Batesian mimicry can fluctuate (change a lot). This mimic ratio is defined as the population of mimics divided by the total population of mimics and the species they are mimicking. Scientists have developed mathematical computer models to identify the optimal (ideal) ratio between mimic and model populations.

Play Along:

Let's find out why the ratio fluctuates with a simplified simulation:

1. Gather small objects in two colors or kinds (marbles, small blocks, cereal, etc..). Select one color to represent an unpalatable species (we'll call these the "Models"). The other color will represent a tasty species that mimics the first species (we'll call these the "Mimics". Remember that the real Mimics and Models would not be different colors – they would look virtually identical.

2. Let's say that in a certain ecosystem, there are more Mimics than Models. Place 60 Mimics in a bag or bowl along with 40 Models. The mimic ratio is: the number of Mimics (60) divided by total number (60+40=100). So the starting ratio is 0.6 or 60% (feel free to use a calculator).

3. Pretend that you are a hungry predator who has never encountered these types of creatures. Reach into the bag or bowl (without looking) to pick your prey (one of the objects). If you pick a Mimic (tasty!), you may pick another "snack" from the bag. If it's also a delicious Mimic, keep picking until you encounter a Model (yuck!). You spit it out and vow to never eat this type of animal again. Remember that the Mimics look so similar to the Models that you (the predator) can't tell them apart, so both Mimics and Models are safe from this individual. How many attempts did it take before you chose an unpalatable Model? What is the current mimic ratio? Record your results in the table below.

4. Without returning the objects to the bag, repeat the simulation with a new predator. In a real population, the survivors would have offspring, but since we're interested in ratios, we'll assume the mimic ratio of the survivors will stay the same and won't add any objects to the bag. Record your results in the table. Repeat for a total of eight predators.

	Starting Ratio	Mimics Eaten	Models Eaten	New Ratio
Predator 1	60.00%			
Predator 2				
Predator 3				
Predator 4				
Predator 5				
Predator 6				
Predator 7				
Predator 8				

Make a line graph of the ratios.

Think about it!

What happened when there was a high mimic ratio?
What about a low mimic ratio?

Chapter 9: To Catch a Thief

Topic 9.1: Genetic fingerprints

In Chapter 9 of "*Math and Magic in Wonderland*", Lulu must identify the thief (the Slithy Tove impostor) in a line-up of Mome Raths. Inspired by a story about mathematician John Napier and his black rooster, she convinces the suspects of the magical powers of a dusty candlestick in the next room before asking them to rub it. Only the true thief, who doesn't want the candlestick to reveal his identity, returns with clean hands.

In our own world, a forensic investigator may compare a fingerprint found at a crime scene with a suspect's fingerprint in order to catch a thief.

Play Along:

Fingerprints are the unique marks left by an individual's fingers. You can take a look at your own fingerprints using a graphite pencil, some clear tape, and a piece of paper.

1. Hold the pencil at an angle and scribble on a small area of paper, making thick black marks over each other to cover the area with graphite.

2. Rub your index finger across the scribbled area to cover your fingertip with graphite.

3. Place you finger on a piece of clear tape, twisting it gently from side to side, but not rubbing it.

4. Stick the tape on a piece of paper to view your unique fingerprint pattern. Classify the pattern you see:

Arch

Loop

Whorl

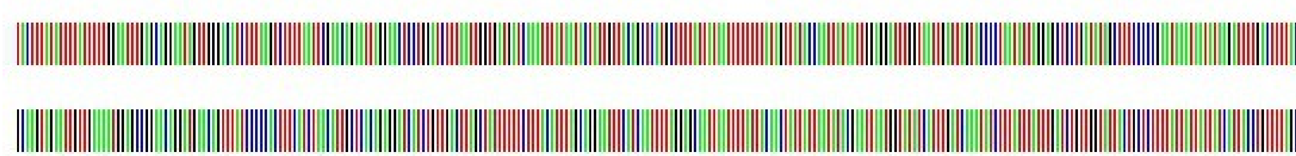
In the animal kingdom, primates and koalas have human-like fingerprints, but many other organisms have unique signatures on their bodies. For example, the stripes on a zebra and the spots on a giraffe make a pattern that is also a one-of-a-kind "fingerprint".

Can you see that each giraffe's pattern is unique? You might remember the "Wild-ID" computer software developed by scientists at Dartmouth University from Chapter 6's activities. Although the software has an 8% error rate for identifying individual wildebeests in photographs, it is virtually flawless at telling giraffes apart.

In the last chapter's activities we discussed different species that are so similar that they are very difficult (and sometimes impossible) to tell apart with the human eye. How can scientists distinguish between these cryptic species? They use genetic fingerprints. DNA, the "blueprint" containing an organism's genetic information, is unique to each individual (except identical twins).

Scientists are collecting DNA barcodes (sections of genetic material) from organisms all over the world and, as a result, are finding new species that were previously thought to be one. There are many useful applications of DNA barcoding. For example, being able to distinguish mosquito species that can carry Malaria (a disease that kills over 400,000 people every year) from those that don't, can aid scientists in combating the disease.

The Monarch and Viceroy butterflies may look similar enough to a predator, but DNA barcodes give away their true identities:



The Monarch's DNA barcode (top) and the Viceroy's (bottom) are quite different.

<u>Play Along:</u>

Try making your own DNA barcodes. DNA is made up of a pattern of four bases – A, T, G, and C. Color the following DNA barcodes using the key. The first one is done for you.

<u>Key</u>
A = Red
T = Blue
G = Green
C = Black

Species 1:

G	G	A	A	C	A	T	G	A	T	G	A	A	C	T

Species 2:

G	G	A	G	G	A	T	T	T	G	G	A	A	T	T

Species 3:

G	A	C	C	G	T	A	G	A	A	T	T	G	C	T

Number of differences between:

Species 1 and Species 2:

Species 1 and Species 3:

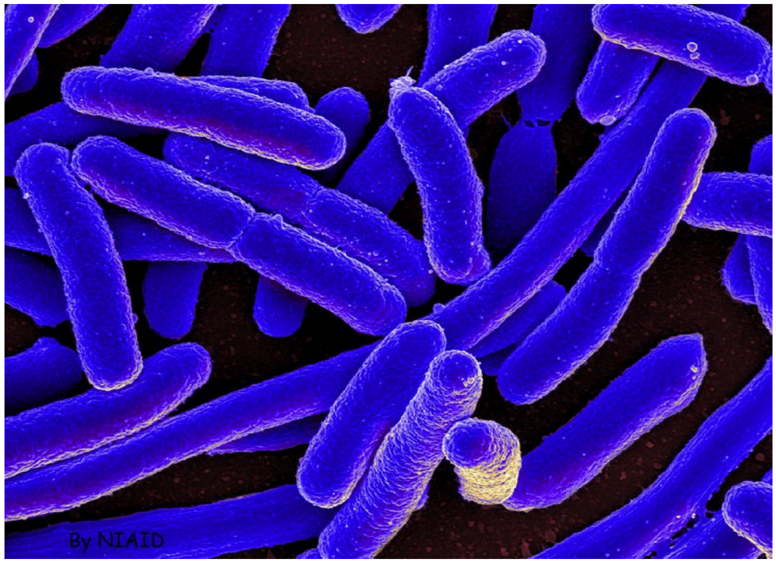
Species 2 and Species 3:

Think about it!

Which two species do you think are related? Why?

Topic 9.2: Exponential growth in bacteria

In Chapter 9 of the novel, Mimsy Kangaroo was shocked at the doubling power of a penny. Let's explore an example of this type of exponential function in nature.



A bacterial population grows by doubling (just like the pennies were doubled each day in the story). The photograph above shows bacteria under a microscope. Can you see some of the bacteria splitting in two? If you start with a single bacterium, it will split into two. Those two bacteria will also split, making a population of four, and so on. This can happen rather quickly under favorable conditions.

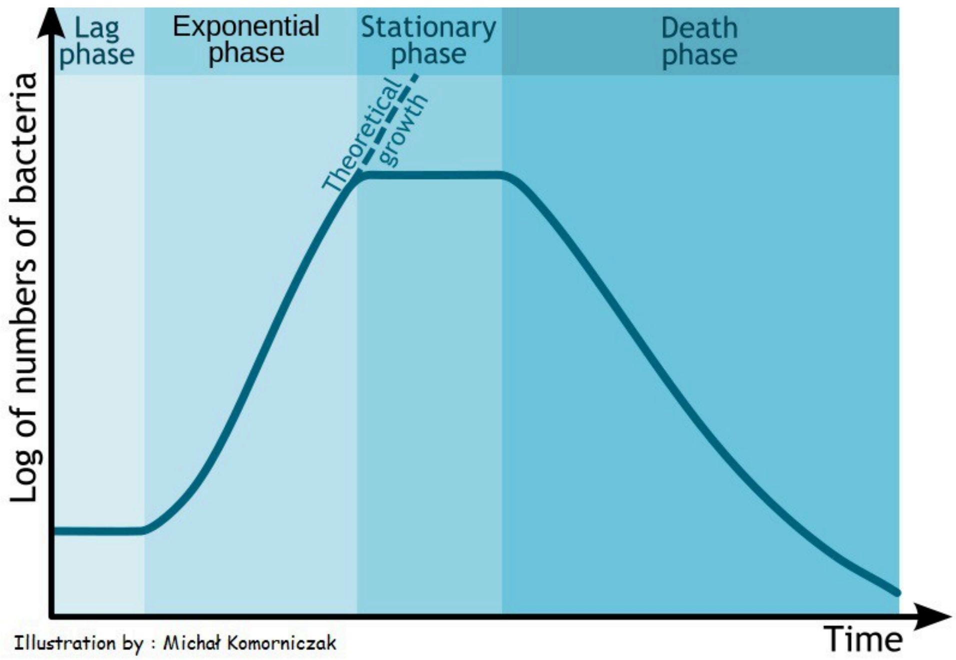
Play Along:

Assume that you have left your sandwich on a picnic table at noon. In the warm sun, this type of bacteria will multiply every ten minutes. Use a calculator to complete the chart below:

Time	Number of Bacteria
12:00	1
12:10	2
12:20	4
12:30	
12:40	
12:50	
1:00	
1:10	
1:20	
1:30	
1:40	
1:50	
2:00	
2:10	
2:20	
2:30	
2:40	
2:50	
3:00	

This sounds like a recipe for food poisoning! Make a line graph with your results on a separate piece of graph paper.

In the real world, bacteria do not keep multiplying forever. They are often limited by certain conditions such as the amount of space or food available. When this limit is reached, the bacterial population will stay the same for a while before the bacteria begin dying rapidly. Here is a diagram of the bacterial life-cycle:



<u>Think about it!</u>

How does the “Exponential phase” in this graph compare to your own line-graph plot?

Chapter 10: The Vorpal Sword

Topic 10.1: Prime numbers and cicadas

"Add the odds to get a square,
Maybe primes are not so rare,
One-one-two-three heeds nature's call
Reveal the thing that conquers all."

The Bandersnatch's riddle held a plethora of math concepts. Let's explore how some of them relate to nature...

A prime number is a number greater than 1 which can only be divided by 1 and itself.

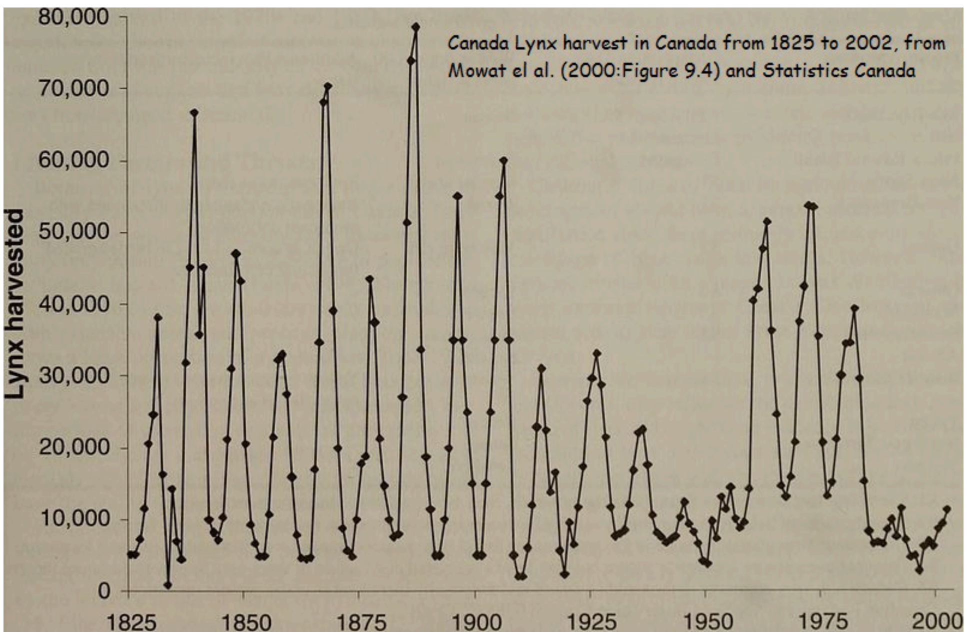
> Think about it!
>
> Gather 13 small objects (like buttons, crayons, or cereal) to verify that 13 is a prime number because it can not be divided into 2, 3, 4, 5, etc.. equal groups. Repeat using 17 objects.

Magicicadas (a genus, or group of species, of cicadas) are insects which spend most of their lives underground and only emerge after 13 or 17 years (to molt, sing loudly, and mate). Is it a coincidence that the length of their life-cycles are prime numbers? In 1977, Harvard scientist Stephen J. Gould published an essay speculating that a life-cycle of this length lowers the number of times that the Cicadas' emergence will coincide with (happen at the same time as) a predator's population cycle.

Population cycles are different from life-cycles. The total population of a species will rise and fall over time. From 1865 to 1875, trappers with the Hudson Bay Company noticed that at first they were catching many snowshoe hare, then the number dropped off dramatically, but later rose again. The trappers identified a 10-year population cycle in the snowshoe hare. Many animals go through these periodic population cycles due to food availability or an increase in predators.

Think about it!

The lynx's primary source of food is the snowshoe hare. Can you predict, how the lynx population is affected by the snowshoe hare's cycle?



When snowshoe hares are plentiful, the Lynx population has enough food and will thrive. When the number of snowshoe hares is low, the Lynx will rely on less nutritional food (such as small rodents) and will not be able to maintain its population.

How is this related to the cicadas? When cicada nymphs emerge from the ground, they have virtually no defense against predators. They are eaten by birds, reptiles, and a variety of mammals. How do they avoid extinction? Cicadas use a strategy called predator satiation. They emerge in such large numbers that they are betting on at least some percentage of the population surviving. This is where the prime-numbered life-cycle comes in.

Play Along:

Most animals have a 2 to 10 year population cycle. Assume that five predators (with different length population cycles) all have population spikes during the same year (year 0). Color the chart to show when their population spikes will occur. The first one is done for you.

Predator with 2-year population cycle: red
Predator with 3-year population cycle: green
Predator with 4-year population cycle: purple
Predator with 6-year population cycle: yellow
Predator with 8-year population cycle: blue

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Now color the cycles for two potential cicada life-cycle lengths:

Cicadas with a life-cycle length of 12 years: black
Cicadas with a life-cycle length of 13 years: pink

<u>Think about it!</u>

What did you notice about the number of predators with an increased population during the years that 12-year cicadas would emerge?

What about the 13-year cicadas would emerge?

Which of these would be more likely to survive as a species?

Thirteen is a prime number, which means that it doesn't have many divisors, so there will be fewer predators that have their population spikes during the same year as the cicadas with the 13-year life-cycle will emerge, giving them a much better chance at survival than if they had a 12-year life-cycle (since 12 has many divisors). This was Stephen J. Gould's hypothesis back in 1977. He further theorized that the prime-numbered life-cycles were a result of evolution (cicadas with life-cycles coinciding with a high population of predators did not survive in large enough numbers to continue the species).

In 2004, a team of Brazilian researchers (Paulo Campos et al.) tested Gould's hypothesis using a computer model. The scientists designed their computer program to model evolution. They started out with predators having random population cycles and prey with random life-cycles and ran the computer program to show what would happen over many generations. In the end, the animals (prey) with prime-numbered life-cycles were the most successful (had the largest populations), demonstrating that this is indeed a result of evolution.

Topic 10.2: Fibonacci numbers, flower petals, and the golden ratio

The pattern "1, 1, 2, 3" should look familiar to you. Do you remember the cabbage's secret code from the activities for Chapter 3? In Fibonacci's sequence, every number is the sum of the previous two numbers:

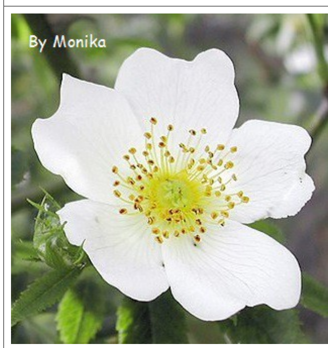
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ….

We've already seen that these numbers are found in spirals, but they can be found in other places, as well.

Play Along:

Count the number of petals on each flower and fill in the chart below (the first row is completed for you):

	Name	Petals	Fibonacci? (Y/N)
	Buttercup	5	Y
	Aster Tataricus		

	Delphinium		
	Aster family		
	Chrysanthemum		
	Wild Rose		
	Lily		

Nature Walk

Go on a nature walk and count petals on flowers. How many were Fibonacci numbers? Sometimes certain species of flowers can have different numbers of petals which average out to a Fibonacci number.

Note that sometimes sepals, which protect the flower as a bud, may be hard to distinguish from true petals (they're usually green, but sometimes the same color as the petals).

In many flowers in the Aster family (like sunflowers) what appear to be the petals surrounding the flower are really “ray flowers”. Some Asters have “disk flowers” on the inside (notice the tiny flowers in the center of a sunflower - how many petals do they have?).

In calculating the height of a tree, we learned about ratios (one length divided by another). Let's explore ratios in the Fibonacci sequence. Use a calculator to fill in the values:

Fibonacci sequence: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610...

A	B	Ratio (B divided by A)
1	2	2.000
2	3	1.500
3	5	1.666
5	8	1.600
8	13	
13	21	
21	34	
34	55	
55	89	
89	144	
144	233	
233	377	
377	610	

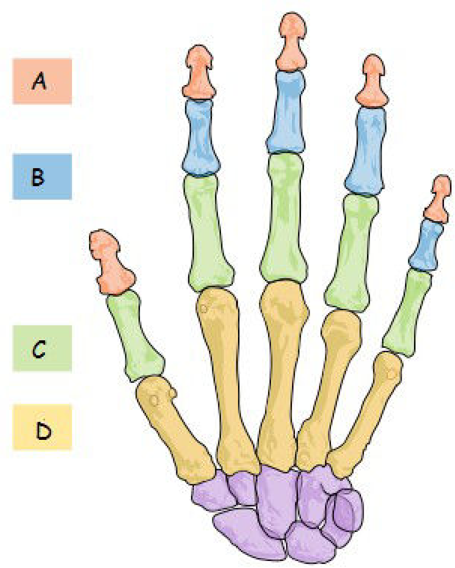
Think about it!

As the numbers get larger in the Fibonacci sequence, their ratios get closer to which number?

In mathematics, when a sequence gets closer and closer to a value, that value is called a limit. The limit of the Fibonacci sequence is called the golden ratio. It is approximately 1.618, called Phi (the Greek symbol ϕ). The golden ratio is found throughout the natural world, as well as in architecture, art, and music. It's even found in your own body...

Play Along:

Measure each section of your index finger (all the way down to your wrist):



Measurements:

A: ________

B: ________

C: ________

D: ________

Ratios:

B divided by A: ________

C divided by B: ________

D divided by C: ________

Think about it!

What did you notice about the ratios between the parts of your hand?

Optional: Here are more ratios on your body to explore:

1. (wrist to elbow) divided by (fingertip to wrist)
2. (bellybutton to ground) divided by (bellybutton to head)
3. (bellybutton to ground) divided by (bellybutton to knees)

Chapter 11: Two Great Powers

Independent Research

In the final chapter of "*Math and Magic in Wonderland"*, Lulu makes a passionate plea of Mrs. Magpie to also make her sister, Elizabeth, into a queen. In her speech, she explains the power of "balancing forces". Nature, too, must maintain balance.

Can you think of examples of how nature is balanced? What happens when human interaction throws off that balance?

Just as Lulu must prove herself worthy of being Queen, now it's your turn to demonstrate the skills you've developed in this guide. Select a topic related to the impact humans are having on our planet and perform some research. Don't forget to follow rabbit trails and see where they lead you.

Dear Reader,

I hope you enjoyed the lessons in this guide. If you haven't had a chance to review "*Math and Magic in Wonderland*" on Amazon or Goodreads, please consider doing so. As a self-published author, I rely on reviews and recommendations to spread the word about my books. Thank you from the bottom of my heart for your feedback and support.

Sincerely,

Lilac Mohr
Author of "*Math and Magic in Wonderland*"