2.2 Newton's Second Law of Motion

EXPLAIN THIS What happens to a car's pickup when you increase your push on it?

Isaac Newton was the first to realize the connection between force and mass in producing acceleration, which is one of the most central rules of nature. He expressed it in his *second law of motion*. **Newton's second law of motion** is

> **The acceleration produced by a net force on an object is directly proportional to the net force, is in the same direction as the net force, and is inversely proportional to the mass of the object.**

Or, in shorter notation,

$$\text{Acceleration} \sim \frac{\text{net force}}{\text{mass}}$$

By using consistent units such as newtons (N) for force, kilograms (kg) for mass, and meters per second squared (m/s^2) for acceleration, we produce the exact equation:

$$\text{Acceleration} = \frac{\text{net force}}{\text{mass}}$$

In briefest form, where a is acceleration, F is net force, and m is mass:

$$a = \frac{F}{m}$$

Acceleration equals the net force divided by the mass. If the net force acting on an object is doubled, the object's acceleration will be doubled. Suppose instead that the mass is doubled. Then the acceleration will be halved. If both the net force and the mass are doubled, then the acceleration will be unchanged. (These relations are nicely developed in the *Conceptual Physical Science Practice Book*.)

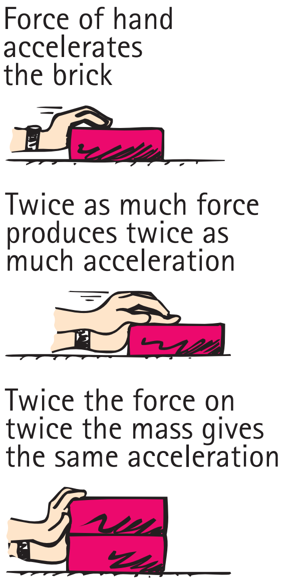


FIGURE 2.7
Acceleration is directly proportional to force.

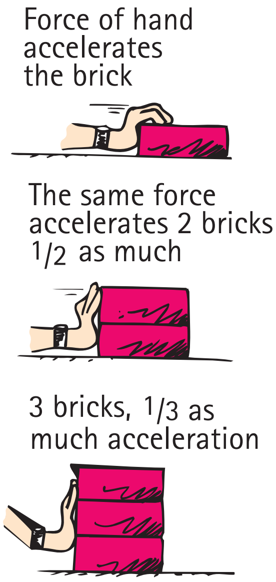
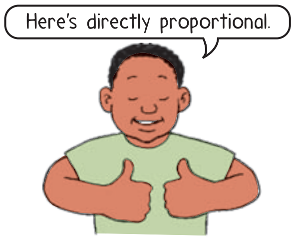
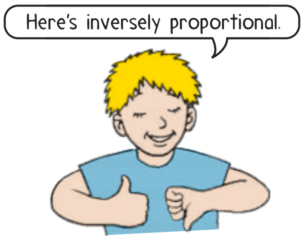


FIGURE 2.8
Acceleration is inversely proportional to mass.





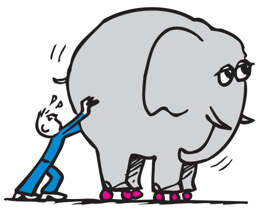


FIGURE 2.6
INTERACTIVE FIGURE MP
Acceleration depends on both the amount of push and the mass being pushed.

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Newton's Second Law

VIDEO:
Force Causes Acceleration

When one thing is inversely proportional to another, as one gets bigger, the other gets smaller.

Force *changes* motion, it doesn't *cause* motion.

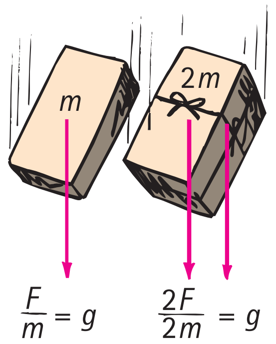

FIGURE 2.9
INTERACTIVE FIGURE MP

The ratio of weight (F) to mass (m) is the same for all objects in the same locality; hence, their accelerations are the same in the absence of air resistance.

CHECKPOINT

1. **In the previous chapter we defined acceleration to be the time rate of change of velocity; that is, a = (change in v)/time. Are we now saying that acceleration is instead the ratio of force to mass—that is, $a = F/m$? Which is it?**
2. **A jumbo jet cruises at constant velocity of 1000 km/h when the thrusting force of its engines is a constant 100,000 N. What is the acceleration of the jet? What is the force of air resistance on the jet?**
3. **Suppose you apply the same amount of force to two separate carts, one cart with a mass of 1 kg and the other with a mass of 2 kg. Which cart will accelerate more, and how much greater will the acceleration be?**

Were these your answers?

1. Both are correct. Acceleration is *defined* as the time rate of change of velocity and is *produced by* a force. How much force/mass (usually the cause) determines the rate change in velocity/time (usually the effect). So we must first define acceleration and then define the terms that produce acceleration.
2. The acceleration is zero, as evidenced by the constant velocity. Because the acceleration is zero, Newton's second law predicts that the net force is zero, which means that the force of air resistance must just equal the thrusting force of 100,000 N and act in the opposite direction. So the air resistance on the jet is 100,000 N. This agrees with $\Sigma F = 0$. (Note that we don't need to know the velocity of the jet to answer this question, but only that it is constant—our clue that acceleration, and therefore net force, is zero.)
3. The 1-kg cart will have more acceleration—twice as much, in fact—because it has half as much mass, which means half as much resistance to a change in its motion.

SCREENCAST: Newton's Second Law

VIDEO: Free Fall Acceleration Explained

When Acceleration Is *g*—Free Fall

Although Galileo founded the concepts of both inertia and acceleration and was the first to measure the acceleration of falling objects, he was unable to explain why objects of various masses fall with equal accelerations. Newton's second law provides the explanation.

We know that a falling object accelerates toward Earth because of the gravitational force of attraction between the object and Earth. As mentioned earlier, when the force of gravity is the only force—that is, when air resistance is negligible—we say that the object is in a state of **free fall.** An object in free fall accelerates toward Earth at 10 m/s^2 (or, more precisely, at 9.8 m/s^2).

The greater the mass of an object, the stronger is the gravitational pull between it and Earth. The double brick in Figure 2.9 for example, has twice the gravitational attraction of the single brick. Why, then, doesn't the double brick fall twice as fast (as Aristotle supposed it would)? The answer is evident in Newton's second law: the acceleration of an object depends not only on the force (weight, in this case), but on the object's resistance to motion—its inertia. Whereas a force produces an acceleration, inertia is a *resistance* to acceleration. So twice the force exerted on twice the inertia produces the same acceleration as half the force exerted on half the inertia. Both accelerate equally. The acceleration due to gravity is symbolized by g. We use the symbol g, rather than a, to denote that acceleration is due to gravity alone.

When Galileo tried to explain why all objects fall with equal accelerations, wouldn't he have loved to know the rule $a = F/m$?

FIGURING PHYSICAL SCIENCE

Problem Solving

If we know the mass of an object in kilograms (kg) and its acceleration in meters per second per second (m/s^2), then the force will be expressed in newtons (N). One newton is the force needed to give a mass of 1 kg an acceleration of 1 m/s^2. We can arrange Newton's second law to read

$$\text{Force} = \text{mass} \times \text{acceleration}$$
$$1\ \text{N} = (1\ \text{kg}) \times (1\ \text{m/s}^2)$$

We can see that

$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$$

The dot between kg and m/s^2 means that the units are multiplied.

If we know two of the quantities in Newton's second law, we can calculate the third.

SAMPLE PROBLEM 1

How much force, or thrust, must a 20,000-kg jet plane develop to achieve an acceleration of 1.5 m/s^2?

Solution:

Using the equation

$$\text{Force} = \text{mass} \times \text{acceleration}$$

we can calculate the force:

$$\begin{aligned} F &= ma \\ &= (20{,}000\ \text{kg}) \times (1.5\ \text{m/s}^2) \\ &= 30{,}000\ \text{kg} \cdot \text{m/s}^2 \\ &= 30{,}000\ \text{N} \end{aligned}$$

Suppose we know the force and the mass, and we want to find the acceleration. For example, what acceleration is produced by a force of 2000 N applied to a 1000-kg automobile? Using Newton's second law, we find that

$$\begin{aligned} a &= \frac{F}{m} = \frac{2000\ \text{N}}{1000\ \text{kg}} \\ &= \frac{2000\ \text{kg} \cdot \text{m/s}^2}{1000\ \text{kg}} = 2\ \text{m/s}^2 \end{aligned}$$

If the force is 4000 N, the acceleration is

$$\begin{aligned} a &= \frac{F}{m} = \frac{4000\ \text{N}}{1000\ \text{kg}} \\ &= \frac{4000\ \text{kg} \cdot \text{m/s}^2}{1000\ \text{kg}} = 4\ \text{m/s}^2 \end{aligned}$$

Doubling the force on the same mass simply doubles the acceleration.

Physics problems are typically more complicated than these.

SAMPLE PROBLEM 2

Here is a more conceptual problem. It is conceptual because it deals not in numbers, but in concepts directly. The focus is showing symbols for concepts, rather than their numerical values. In the next sample problem, force is *F*, mass is *m*, and acceleration is *a*. This way you build a habit of first thinking in terms of concepts and the symbols that represent them. Part (b) follows up and brings in the numbers after you've done the physics.

A force *F* acts in the forward direction on a carton of chocolates of mass *m*. A friction force *f* opposes this motion.
(a) Use Newton's second law and show that the acceleration of the carton is

$$\frac{F - f}{m}$$

(b) If the carton's mass is 4.0 kg, the applied force is 12.0 N, and the friction force is 6.0 N, show that the carton's acceleration is 1.5 m/s^2.

Solution:

(a) We're asked to find the acceleration. From Newton's second law we know that $a = (F_{net})/m$. Here the net force is $F - f$. So the solution is $a = (F - f)/m$ (where all quantities represented are known values). Notice that this answer applies to all situations in which a steady applied force is opposed by a steady frictional force. It covers many possibilities.
(b) Here we simply substitute the numerical values given:

$$\begin{aligned} a &= \frac{F - f}{m} = \frac{12.0\ \text{N} - 6.0\ \text{N}}{4.0\ \text{kg}} \\ &= 1.5 \frac{\text{N}}{\text{kg}} = 1.5\ \text{m/s}^2 \end{aligned}$$

(The units N/kg are equivalent to m/s^2.) Note that the answer, about 15% of *g*, is "reasonable." For more on units of measurement and significant figures, see your Lab Manual.

The ratio of weight to mass for freely falling objects equals the constant *g*. This is similar to the constant ratio of circumference to diameter for circles, which equals the constant π. The ratio of weight to mass is identical for both heavy and light objects, just as the ratio of circumference to diameter is the same for both large and small circles (Figure 2.10).

We now understand that the acceleration of free fall is independent of an object's mass. A boulder 100 times as massive as a pebble falls at the same acceleration as the pebble because although the force on the boulder (its weight) is 100 times the force (or weight) on the pebble, its resistance to a change in motion (mass) is 100 times that of the pebble. The greater force offsets the correspondingly greater mass.

Ironically, Galileo couldn't say *why* all bodies fall equally because he never connected the concepts he developed—*acceleration* and *inertia*—with *force*. That connection awaited Newton's second law.

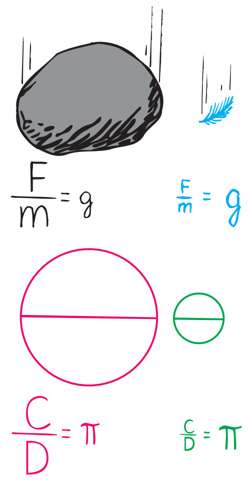

FIGURE 2.10